TRINITY COLLEGE

Year 12 Mathematics Specialist
Test 6 2019

Section 2 Calculator Assumed

Rectilinear Motion, Simple Harmonic Motion and Statistical Inference

STUDENT'S NAME _____

DATE: Monday 9 September **TIME:** 50 minutes **MARKS**: 50

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, formula page

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Mr Presser samples 50 year 12 students. The mean height, μ of the sample is 172 cm and the standard deviation, σ is 11 cm.

Determine a 98% confidence interval for the sample mean.

2. (9 marks)

Mr Jamieson wants to estimate the population mean number of minutes, μ, that male year 12 students read per week. He takes a random sample of 73 students and determines a 95% confidence interval for μ. The upper limit of this interval is 47.22 minutes and the lower limit is 42.78 minutes.

(a) Determine the sample mean for this sample of 73 students. [2]

(b) Calculate, correct to 0.01 minutes, the sample standard deviation for the sample of 73 students. [3]

(c) A student makes the following statements. Explain why each statement is either true or false.

(i) The probability that the sample mean lies in this confidence interval is 95% [2]

(ii) The amount of time that students spend reading is normally distributed because a distribution of sample means is normally distributed. [2]

3. (10 marks)

The mean wait time at a set of traffic signals has been observed to be normally distibuted with a mean $\mu = 60$ seconds and standard deviation $\sigma = 20$ seconds.

The wait times are recorded 100 times. Determine the probability that the:

(a) sample mean wait time will be between 57 seconds and 63 seconds. [3]

Accuracy is required by the department managing the traffic signals. The mean wait time at the traffic signals is recorded a number of times.

(b) If the probablity for the mean wait time differs from μ by less than 5 seconds is 96%, determine n , the number of wait times that need to be measured. [3]

Daivik decides to test the validity of the department's claimed mean wait time. He records ten wait times on a stopwatch for a total time of 12 minutes. Daivik states that 'the departments figures for the average wait time are correct!'

(c) Perform the calculations necessary to comment on this claim. [4]

4. (10 marks)

If, at any time t , a particle moves along a straight line with acceleration a cm/s^2, velocity v cm/s and displacement x cm from a fixed point O , the relationship between these variables is given by

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

(a) Use the chain rule to show that $\frac{d}{dx}(\frac{1}{2}v^2) = a$ [2]

For the case when $a(t) = e^{2x}$ with initial conditions $x(0) = 1$ cm and $v(0) = e$ cm/s, determine

(b) the velocity of the particle when $x(t) = 5$ cm. [4]

(c) the time when the particle will have a displacement of 10 cm to the right of the origin.

[4]

5. (11 marks)

A particle moves along a straight line and its distance x metres from a fixed point O on the line after t seconds is given by

$$x = B\sin(kt - \theta)$$

where B , k and θ are positive constants and $\theta < 2\pi$.

The particle passes away through the point O for the first time after 2 seconds and away for the second time after 7 seconds. The maximum distance that the particle moves away from the point O is 10 metres.

(a) Determine the values of B , k and θ [5]

(b) When is the first time that the particle is furthest away from O ? [3]

(c) What is the maximum speed of the particle? [3]

6. (7 marks)

The displacement, $x(t)$ metres, of an object undergoing rectilinear motion is given by

$$x(t) = A\cos\omega t + B\sin\omega t$$

(a) Show that the object is undergoing simple harmonic motion. [2]

Initially, the object is located at $x = 1\,\text{m}$ and has velocity $v = 3\,\text{m/s}$ with period π metres.

(b) Show that at time t, the position of the object is $x = \cos 2t + \frac{3}{2}\sin 2t$ [3]

(c) Determine when the object first has maximum velocity. [2]